लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 4th Semester Supplementary Examination, 2021

MTMHGEC04T/MTMGCOR04T-MATHEMATICS (GE4/DSC4)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Write down the power set of the set $\{x,\ y,\ z\}$.

(b) Two mappings $f:R\to R$ and $g:R\to R$ are defined by $f(x)=x^2$; $g(x)=2x+3$ for all $x\in R$. Find the mapping $f\circ g$.

(c) Let $f=\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 3 & 4 & 2\end{pmatrix}$ and $g=\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 4 & 1\end{pmatrix}$ be two permutations. Then find gf.

(d) Prove that $(Z,\ \cdot)$ is not a group, where Z is the set of integers.

(e) State Lagrange's theorem.

(f) Find the generators of the cyclic group $\{\mathbb{Z}, +\}$.

(g) Prove that under the matrix addition and multiplication, the set $M=\left\{\begin{pmatrix}\alpha & \alpha\\ 0 & \alpha\end{pmatrix}:\ \alpha\in Z\right\}$ is not a ring.

(h) Prove that if every element of a group $(G,\circ)$ be its own inverse then it is an abelian group.

2. (a) Show that a group $(G,*)$ is commutative if and only if $(a*b)^2=a^2*b^2$ for all $a\in G$ [where $x^2=x*x$]. 5

(b) Let $(G,\circ)$ be a group. Prove that $(a*b)^{-1}=a^{-1}*b^{-1}$ for all $a,b\in G$. 3

3. (a) Let $(G,\circ)$ be a group. A non-empty subset H of G forms a subgroup of $(G,\circ)$ if and only if $a\in H,\ b\in H\Rightarrow a\circ b^{-1}\in H$. 4

(b) Let $S=\{1,\ \omega,\ \omega^2\}$, where ω is an imaginary cube root of 1. Prove that S is an abelian group with respect to multiplication. 4

4. (a) Show that the set of even integers forms a commutative ring with respect to the usual addition and multiplication of integers. 5

(b) If a ring $(R, +, \cdot)$, $a^2 = a$ for all $a \in R$; prove that $a + a = 0$ for all $a \in R$; (0 is the zero element of R). 3

5. (a) In a ring $(R, +, \cdot)$ show that $(-a)\cdot(-b) = a \cdot b$ for all $a, b \in R$. 3

(b) Let $M = \left\{ \begin{pmatrix} 2a & 0 \\ 0 & 2b \end{pmatrix} : a, b \in Z \right\}$, then show that M is a ring with respect to matrix addition and multiplication. 5

6. (a) Prove that if G is commutative, then every subgroup of G is normal. 4

(b) Let G be a group and H be a subgroup of G. If $h \in H$ then prove that $hH = H$. 4

7. (a) Prove that every proper subgroup of a group of order 6 is cyclic. 4

(b) The intersection of two normal subgroups of a group G is a normal subgroup of G. 4

8. (a) Show that $H = \left\{ \begin{pmatrix} a+ib & c+id \\ -c+id & a-ib \end{pmatrix} : a, b, c, d \in Z \right\}$ forms a ring with unity. 4

(b) Prove that a finite integral domain is a field. 4

9. (a) If f is real function defined by $f(x) = \dfrac{x-1}{x+1}$, then prove that $f(2x) = \dfrac{3f(x)+1}{f(x)+3}$. 3

(b) If $f(x) = x^2$, then find the value of $\dfrac{f(1.1) - f(1)}{1.1 - 1}$. 2

(c) Prove that a group $(G, \circ)$ contains only one identity element. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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